A direct measurement of the generating function for photoelectron counting experiments in the case of a Lorentz spectral line



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# Letters to the Editor

## A direct measurement of the generating function for photoelectron counting experiments in the case of a Lorentz spectral line†

**Abstract.** Measurements of the probability $P(0)$ of detecting zero optical photons during a fixed counting time are used to verify an exact calculation of this quantity for the case of gaussian light with a Lorentz spectral line. The measurement of $P(0)$ is of particular interest since it provides an explicit measurement of the 'generating function' $Q(\lambda, t)|_{\lambda=1}$ used to generate all of the other probabilities $P(n)$. The excellent agreement of the measurements with the exact calculation validates the use of this calculation in determining the corrections to the 'short time' approximation commonly used in photon counting experiments.

Recent advances in fast pulse electronics have made the use of photon counting devices practical for the measurement of the spectra of light which are characterized by relatively narrow linewidths. One quantity which is of considerable interest in such experiments is $P(n, T, n)$ which we define to be the probability of detecting $n$ photons in a time $T$ when the average photon count rate is $n$. These probabilities can be displayed most conveniently by using the 'generating function' $Q$ which is defined (Glauber 1965) as follows:

$$Q(\lambda, T, n) \equiv \sum_{n=0}^{\infty} (1-\lambda)^n P(n, T, n). \tag{1}$$

If the light under examination is characterized by a Lorentz spectrum whose half-width is $\Gamma$, this generating function can be computed exactly for all values of $n$ and $T$ (Jakeman and Pike 1968, Barakat and Glauber 1971, to be published).

$$Q(\lambda, \tau, v) = e^{\tau}\{\cosh(\tau\beta) + (\tfrac{1}{2}\beta)(\beta^2+1)\sinh(\tau\beta)\}^{-1}. \tag{2}$$

In the above equation we have followed the notation of Barakat and Glauber (1971, to be published) and written the generating function in terms of the dimensionless quantities $\tau \equiv \Gamma T$ and $v \equiv n/\Gamma$. We have also defined

$$\beta \equiv (1+2\lambda v)^{1/2}. \tag{3}$$

When $\tau$ is small, that is, when the counting time $T$ is small compared with the time $1/\Gamma$, the generating function can be approximated as (Bedard 1967)

$$Q(\lambda, \tau, v) \simeq (1+v\lambda\tau)^{-1} = (1+\lambda n T)^{-1}. \tag{4}$$

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Returning to equation (1) we see that if we measure the probability $P(0, T, n)$ that zero photon counts are detected during the time $T$, we have also measured the generating function at $\lambda = 1$

$$P(0, \tau, v) = Q(1, \tau, v). \tag{5}$$

In figure 1 we illustrate the instrument which we have used to measure $P(0, \tau, v)$.

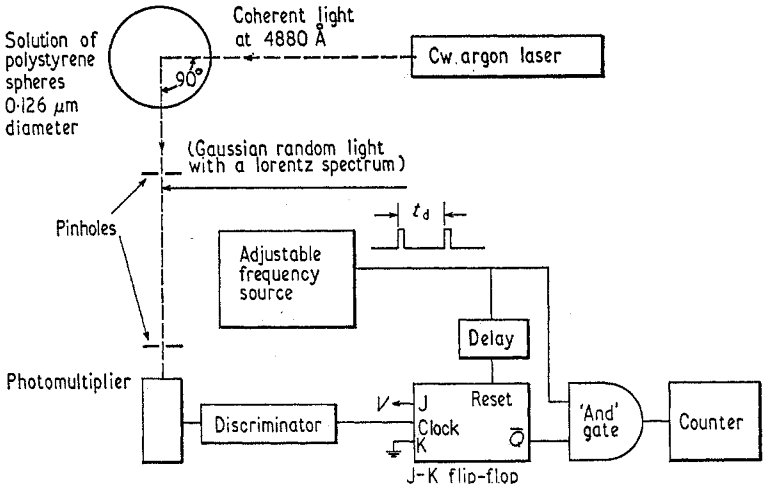


Figure 1. Zero count detector.

Light with a Lorentz spectrum is produced by scattering coherent laser light with a wavelength of 4880 Å from a suspension of polystyrene particles which are 0·126 μm in diameter. The linewidth Γ produced by this scattering process was measured by using a photon correlation device (Jakeman *et al.* 1968). In this way we measured $\Gamma = (2{\cdot}29 \pm 0{\cdot}04) \times 10^3 \text{ s}^{-1}$. We could also use the photon correlation instrument to determine if the light incident on our detectors contained any nonlorentzian components. These components constituted less than 1% of the intensity of the incident light.

The scattered light was passed through two pinholes so that only a fraction of a 'coherence area' (Jakeman *et al.* 1970, Kelly 1972) illuminated the photocathode of our photomultiplier (we used a Bendix 754 'photon counter'). The output of this tube was amplified and standardized by a discriminating system and the resulting standardized pulses were then presented to the 'zero pulse detector' shown in figure 1. This device sent a pulse to the counter whenever zero counts were received from the discriminator during the time $T$.

We measured $P(0)$ by simply recording the number of 'zero pulse' counts produced during a time $T_0$ and dividing this number by $T_0/T$. The results of our experiments for various values of $\tau$ and $v$ are shown in figure 2. The full curves in this figure are calculated from the measured values of Γ and $n$ using equation (2) while the broken curves represent the 'short counting time' approximation of equation (4).

The experiments show that the departures from the short time approximation are accurately explained by the exact lorentzian result. We believe that the small deviation from the exact result at high count rates is probably due to the finite pulse widths used by our equipment and by 'jitter' in the zero pulse detector.

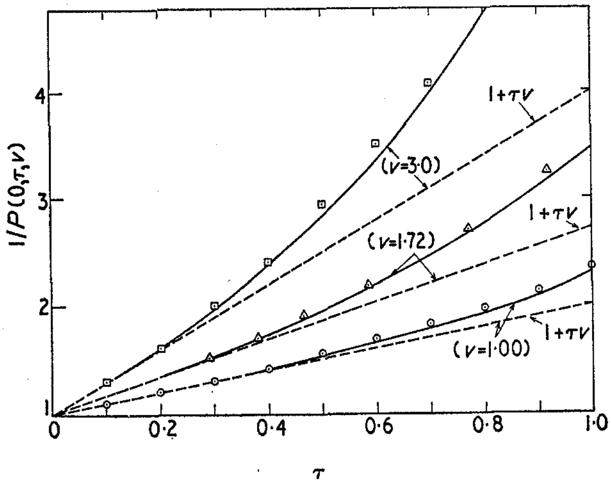


Figure 2. Theoretical and experimental plots of $1/P(0, \tau, v)$ against $\tau$. ⊙ Measured for $v = 1{\cdot}00$; △ measured for $v = 1{\cdot}72$; □ measured for $v = 3{\cdot}00$.

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Bedard, G., 1967, *Phys. Rev.*, **161**, 1304–8.
Glauber, R. J., 1965, *Quantum Optics and Electronics*, eds C. DeWitt, A. Glandin and C. Cohen (New York: Gordon and Breach), p. 178.
Jakeman, E., Oliver, C. J., and Pike, E. R., 1968, *J. Phys.* A: *Gen. Phys.*, **1**, 406–8.
—— 1970, *J. Phys.* A: *Gen. Phys.*, **3**, L55–9.
Jakeman, E., and Pike, E. R., 1968, *J. Phys.* A: *Gen. Phys.*, **1**, 128–38.
Kelly, H. C., 1972, *J. Phys.* A: *Gen. Phys.*, in the press.

# The effect of atomic excitation on the shape of the $^3H$ beta spectrum near the end point

**Abstract.** The effect of atomic excitation on the shape of the $^3H$ beta spectrum near the end point is calculated and found to be small. However, depending on the type of spectrometer and the method of analysis of data, the effect could lead to a systematic error which is not negligible compared with the accuracies quoted in some recent determinations of the end point.

The beta decay of $^3H$ is the simplest allowed decay, apart from the decay of the free neutron, and accurate study of the spectrum near the end point is important for